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Preface

The world of computing and communication has reached a level of visual content that was hard to imagine even ten years ago. Browsers download images, applets create animated sequences; entire movies are made with 100% computer-generated 3D imagery. Several of the underlying computational issues have their home in the field of *Computer Aided Geometric Design*, or CAGD. *The Essentials of CAGD* is an elementary introduction to those concepts which can be used to model the letters in this book as well as the “actors” in *Toy Story*.¹

CAGD goes back to the 1950s when computers were used to drive numerically controlled milling machines in the automotive and aircraft industries. The basic tools that were developed then are *parametric curves and surfaces*. Now these are not only used in design and manufacturing (CAD/CAM) but also in computer graphics, computer animation, 3D visualization, reverse engineering, or robotics.

Several texts exist on the topic of CAGD. Why a new one? Some texts, for example Farin [9] or Hoschek/Lasser [15] assume a level of mathematical sophistication that is, in our experience, overwhelming for novices. Texts at lower math levels typically miss out on applications. The CAGD coverage in Computer

¹An animated movie produced by Pixar Studios.

Graphics texts is spotty. Hence we tried to create a comprehensive introduction that addresses a general audience and that covers many applications.

The Essentials of CAGD is intended for anyone who needs to learn the basic concepts of CAGD, be it as a first-time student or as a practitioner whose skills are a bit rusty. Its theoretical level is kept as low as possible – we usually substitute examples and images for exact proofs. *The Essentials of CAGD* is meant to be used at the freshman/sophomore undergraduate level. It serves as an introduction to CAGD for engineers or computer scientists. It is also an ideal companion text for a computer graphics class. Prerequisites for this text include basic computer graphics and linear algebra (such as provided in [10]).

The Essentials of CAGD approaches each topic from a geometric viewpoint. This is realized in three ways:

- **Sketches** illustrate the geometric elements of a concept.
- **Figures** illustrate a computer application of a concept.
- **Examples** illuminate algorithms by stepping the reader through a numerical application of a concept.

Exercises are listed at the end of each chapter. Solutions to selected exercises are given in Appendix 13.9.

There is a short Bibliography, suggesting texts for reference and more advanced study of CAGD and related topics. The field of CAGD has its own journal – visit www.elsevier.nl/locate/comaid.

The Essentials of CAGD has a web site:
<http://eros.eas.asu.edu/~farin/essbook/essbook.html>.

This web site contains general information and updates. It also contains most PostScript files used in the book. See the web page for details on downloading these files. Also available are all data files referred to in the text. In the near future, the site will contain an errata page.

We like to thank the members of Arizona State University's PRISM project.² Particular thanks go to Mary Zhu for help with many graphics problems. Also thanks to M-S. Bae, J. McIntosh, A. Nasri, A. Razdan, H. Theisel. As usual, it was a pleasure to work with AK Peters during all stages of the publication process.

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²For more info on this interdisciplinary project, visit <http://surdas.eas.asu.edu/prism/prism/>.

